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THREE-DIMENSIONAL MULTIPLE BEAM ANALYSIS OF A SATURN I VEHICLE

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ABSTRACT

A normal mode vibration analysis was made of a Saturn I vehicle using a newly developed lumped parameter multiple beam representation incorporating coupling of the center tank motion in pitch, yaw, and roll with the motions of the outer tanks. The analysis demonstrates the uncoupling of motions in pitch, yaw, and roll for a vehicle having a symmetrical distribution of mass and stiffness. A few cases of nonsymmetry were examined and numerical results were obtained which show the effect of vehicle nonsymmetry on the natural frequencies of the vehicle.

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STRUCTURAL DYNAMICS SECTION
DYNAMICS ANALYSIS BRANCH
DYNAMICS AND FLIGHT MECHANICS DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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DEFINITION OF SYMBOLS

Symbol	Definition
$\vec{D}_c(x)$	total displacement of center tank
$\vec{J}$, $\vec{K}$, $\vec{\Psi}$	unit vector for center tank in pitch, yaw, and roll, respectively
$\vec{r}_T$, $\vec{t}_T$	unit vector for outer tanks, radial and tangential, respectively
$\eta_i(t)$	i^{th} mode amplitude factor for center tank bending in pitch
$\mu_i(t)$	i^{th} mode amplitude factor for center tank bending in yaw
$y_{ci}(x)$	i^{th} mode deflection of center tank in pitch
$z_{ci}(x)$	i^{th} mode deflection of center tank in yaw
$\eta_y(t)$	amplitude factor for center tank rigid body translation in pitch
$\mu_z(t)$	amplitude factor for center tank rigid body translation in yaw
Y_c	rigid body translation of center tank in pitch
Z_c	rigid body translation of center tank in yaw
$\bar{x}_c(x)$	$x - x_{mc}$, where x_{mc} is the center of mass of center tank
$\eta_\phi(t)$	amplitude factor for center tank rigid body rotation in pitch
$\mu_\theta(t)$	amplitude factor for center tank rigid body rotation in yaw
ϕ_c	rigid body rotation of center tank in pitch
θ_c	rigid body rotation of center tank in yaw

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
$\nu_f(t)$	amplitude factor for center tank twisting in roll
$\nu(t)$	amplitude factor for center tank rigid body rotation in roll
$\psi_f(x)$	torsional deformation of center tank in roll
ψ	rigid body rotation of center tank in roll
$\vec{D}_T(x)$	total displacement of outer tank
$\lambda_{fT}(t)$	amplitude factor for outer tank radial bending
$\sigma_{fT}(t)$	amplitude factor for outer tank tangential bending
$r_{Tf}(x)$	deflection of outer tank in radial bending
$t_{Tf}(x)$	deflection of outer tank in tangential bending
$\lambda_{RT}(t)$	amplitude factor for outer tank rigid body motion, radial
$\sigma_{RT}(t)$	amplitude factor for outer tank rigid body motion, tangential
R_T	rigid body radial displacement of outer tank
T_T	rigid body tangential displacement of outer tank
$\lambda_{\gamma T}(t)$	amplitude factor for rigid body rotation of outer tank - radial motion
$\sigma_{\beta T}(t)$	amplitude factor for rigid body rotation of outer tank - tangential motion
γ_T	rigid body rotation of outer tank - radial motion

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
β_T	rigid body rotation of outer tank - tangential motion
$\bar{x}_T(x)$	$x - x_{mT}$, where x_{mT} is the center of mass of outer tank
$\bar{m}_c(x)$	mass per unit length of center tank
$\bar{m}_T(x)$	mass per unit length of outer tank
ℓ_c	length of center tank
ℓ_T	length of outer tank
$\bar{I}_{\nu c}(x)$	rotary inertia per unit length of center tank
R	distance from C.G. of center tank to C.G. of outer tank
$\vec{\Delta}_1(x)$	motion of center tank with respect to outer tank 1
a	station at top connection of center and outer tanks
b	station at lower connection of center and outer tanks
s	station at suspension point of center tank
$y_c^{*\prime}(x)$, $z_c^{*\prime}(x)$	slope in pitch and yaw, respectively
k_{LJ}, k_{LK}	longitudinal spring constant of outer tank in pitch and yaw, respectively
k_{SJ}, k_{SK}	translational suspension spring constant in pitch and yaw, respectively
k_{SRJ}, k_{SRK}	rotational suspension spring constant in pitch and yaw, respectively
k_ψ	torsional suspension spring constant
k_{aTr}, k_{bTr}	radial connection spring constants of outer tank at a and b, respectively

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
k_{aTt}, k_{bTt}	tangential connection spring constants for outer tank at a and b, respectively
k_{rTr}	radial rotational connection spring constant for outer tank
k_{rTt}	tangential rotational connection spring constant for outer tank
$r_T^{*\prime}(x)$	slope of outer tank in radial motion
$t_T^{*\prime}(x)$	slope of outer tank in tangential motion
$\zeta(T)$	+1 for T = 1, 3, 6, 8; -1 for T = 2, 4, 5, 7
M_{GY}, M_{GZ}	center tank generalized mass in pitch and yaw, respectively
ω_{FFY}, ω_{FFZ}	uncoupled natural frequency in pitch and yaw, respectively
$I_{G\psi}$	generalized torsional inertia of center tank
$\omega_{FF\psi}$	uncoupled natural frequency in torsion
M_{GTr}, M_{GTt}	generalized mass of outer tank, radial and tangential, respectively
ω_{HHTr}, ω_{HHTt}	uncoupled outer tank natural frequency, radial and tangential, respectively
ω	coupled natural frequency of complete system

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THREE-DIMENSIONAL MULTIPLE BEAM ANALYSIS OF A SATURN I VEHICLE

SUMMARY

A normal mode vibration analysis was made of a Saturn I vehicle using a newly developed lumped parameter multiple beam representation incorporating coupling of the center tank motion in pitch, yaw, and roll with the motions of the outer tanks. The analysis demonstrates the uncoupling of motions in pitch, yaw, and roll for a vehicle having a symmetrical distribution of mass and stiffness. A few cases of non-symmetry were examined and numerical results were obtained which show the effect of vehicle nonsymmetry on the natural frequencies of the vehicle.

I. INTRODUCTION

This report presents a normal mode vibration analysis of a complex space vehicle of the Saturn I type. The analysis uses a lumped parameter multiple beam representation of the vehicle which incorporates the coupling of motion of the center tank pitch, yaw, and torsion with the motions of the outer tanks. The outer tank motions include the radial and tangential degrees of freedom of each of the eight outer tanks.

The analysis demonstrates the uncoupling of motions in pitch, yaw, and torsion for a vehicle having a symmetrical distribution of mass and stiffness. The solution presented is generally capable of treating most types of nonsymmetry; however, the limitation of available computer programs prevented treating more complicated nonsymmetries than those treated in this presentation.

II. MATHEMATICAL MODEL

The Saturn I vehicle consists of a booster center tank to which are attached, by various connecting members, four outer LOX tanks and four outer fuel tanks; above the booster are the upper stages of the vehicle. The vehicle structure and configuration are shown in the reference.

In the multiple beam type of analysis, the center tank of the booster and the upper stages are considered as a single free-free beam. The outer tanks are treated as separate beams elastically connected to the center beam. More detail regarding the assumptions involved in this representation can be found in the reference.

A schematic section of the booster stage showing the unit vectors defined for the coupled system of equations developed in the analysis is shown in Figure 1. All vectors are unit length.

III. EQUATIONS OF MOTION

The motion of the center tank was represented by three flexible modes and two rigid body modes in both the pitch and yaw directions, and by one flexible mode and one rigid body mode in the torsional degree of freedom. Thus, displacement of the center tank is

$$\vec{D}_c(x) = \eta^*(t)\ y_c^*(x)\ \vec{J} + \mu^*(t)\ z_c^*(x)\ \vec{K} + \nu^*(t)\ \psi^*(x)\ \vec{\psi}$$

where

$$\eta^*(t)\ y_c^*(x) = \sum_{i=1}^{3} \eta_i(t)\ y_{ci}(x) + \eta_y(t)\ Y_c - \bar{x}_c(x)\ \eta_\phi(t)\ \phi_c$$

$$\mu^*(t)\ z_c^*(x) = \sum_{i=1}^{3} \mu_i(t)\ z_{ci}(x) + \mu_z(t)\ Z_c - \bar{x}_c(x)\ \mu_\theta(t)\ \theta_c$$

$$\nu^*(t)\ \psi^*(x) = \nu_f(t)\ \psi_f(x) + \nu(t)\ \psi.$$

The motion of each outer tank was represented by one flexible mode and two rigid body modes in both the radial and tangential directions. Thus, displacement of the outer tanks is

$$\vec{D}_T(x) = \lambda_T^*(t)\ r_T^*(x)\ \vec{r}_T + \sigma_T^*(t)\ t_T^*(x)\ \vec{t}_T; \qquad T = 1, 2, \ldots, 8$$

where

$$\lambda_T^*(t)\ r_T^*(t) = \lambda_{fT}(t) r_{Tf}(x) + \lambda_{RT}(t)\ R_T - \bar{x}_T(x)\ \lambda_{\gamma T}(t)\ \gamma_T;$$

$$T = 1, 2, \ldots, 8$$

$$\sigma_T^*(t)\ t_T^*(x) = \sigma_{fT}(t)\ t_{Tf}(x) + \sigma_{RT}(t)\ T_T - \bar{x}_T(x)\ \sigma_{\beta T}(t)\ \beta_T;$$

$$T = 1, 2, \ldots, 8.$$

The total kinetic energy of the system is

$$\text{K.E.} = \frac{1}{2} \int_{\ell_c} \bar{m}_c(x) \left\{ \left[\frac{d}{dt}\ \eta^*(t)\ y_c^*(x)\right]^2 + \left[\frac{d}{dt}\ \mu^*(t)\ z_c^*(x)\right]^2 \right\} dx$$

$$+ \frac{1}{2} \int_{\ell_c} \bar{I}_{\nu c}(x) \left[\frac{d}{dt}\ \nu^*(t)\ \psi^*(x)\right]^2 dx$$

$$+ \sum_{T=1}^{8} \frac{1}{2} \int_{\ell_T} \bar{m}_T(x) \left\{ \left[\frac{d}{dt}\ \lambda_T^*(t)\ r_T^*(x)\right]^2 + \left[\frac{d}{dt}\ \sigma_T^*(t)\ t_T^*(x)\right]^2 \right\} dx.$$

To compute the potential energy of the system, the center tank is related to the coordinate systems of each of the outer tanks by transformations, one of which is given here as an example:

Tank I: $\vec{K} = \frac{\sqrt{2}}{2}\ \vec{r}_1 + \frac{\sqrt{2}}{2}\ \vec{t}_1$

$$\vec{J} = -\frac{\sqrt{2}}{2}\ \vec{r}_1 + \frac{\sqrt{2}}{2}\ \vec{t}_1$$

$$\vec{\psi} = -\ R\vec{t}_1$$

$$\vec{D}_c(x) = \left[-\frac{\sqrt{2}}{2}\vec{r}_1 + \frac{\sqrt{2}}{2}\vec{t}_1\right]\left[\eta^*(t)\ y_c^*(x)\right] + \left[\frac{\sqrt{2}}{2}\vec{r}_1 + \frac{\sqrt{2}}{2}\vec{t}_1\right]\left[\mu^*(t)\ z_c^*(x)\right]$$

$$- R\nu^*(t)\ \psi^*(x)\ \vec{t}_1$$

$$\vec{\Delta}_1(x) = \vec{r}_1\left[\lambda_1^*(t)\ r_1^*(x) + \frac{\sqrt{2}}{2}\eta^*(t)\ y_c^*(x) - \frac{\sqrt{2}}{2}\mu^*(t)\ z_c^*(x)\right]$$

$$+ \vec{t}_1\left[\sigma_1^*(t)\ t_1^*(x) - \frac{\sqrt{2}}{2}\eta^*(t)\ y_c^*(x) - \frac{\sqrt{2}}{2}\mu^*(t)\ z_c^*(x)\right.$$

$$\left. + R\nu^*(t)\ \psi^*(x)\right].$$

The total potential energy of the system is

$$\text{P.E.} = \frac{1}{2}k_{LJ}\left[\eta^*(t)\ y_c^{*\prime}(a) - \eta^*(t)\ y_c^{*\prime}(b)\right]^2 + \frac{1}{2}k_{LK}\left[\mu^*(t)\ z_c^{*\prime}(a) -\right.$$

$$\left.- \mu^*(t)\ z_c^{*\prime}(b)\right]^2 + \frac{1}{2}k_{SJ}\left[\eta^*(t)\ y_c^*(s)\right]^2 + \frac{1}{2}k_{SK}\left[\mu^*(t)\ z_c^*(s)\right]^2$$

$$+ \frac{1}{2}k_{SRJ}\left[\eta^*(t)\ y_c^{*\prime}(s)\right]^2 + \frac{1}{2}k_{SRK}\left[\mu^*(t)\ z_c^{*\prime}(s)\right]^2 + \frac{1}{2}k_{\psi}\left[\nu^*(t)\ \psi^*(s)\right]^2$$

$$+ \sum_{T=1,2,5,6}\frac{1}{2}k_{aTr}\left[\lambda_T^*(t)\ r_T^*(a) - (-1)^T\frac{\sqrt{2}}{2}\eta^*(t)\ y_c^*(a)\right.$$

$$\left.- \frac{\sqrt{2}}{2}\mu^*(t)\ z_c^*(a)\right]^2$$

$$+ \sum_{T=3,4,7,8} \frac{1}{2} k_{aTr} \left\{ \lambda_T^*(t)\ r_T^*(a) - \frac{1}{2}\left[(-1)^T + 1\right] \eta^*(t)\ y_c^*(a) \right.$$

$$\left. + \frac{1}{2}\left[(-1)^T - 1\right] \mu^*(t)\ z_c^*(a) \right\}^2 + \sum_{T=1,2,5,6} \frac{1}{2} k_{bTr} \left[\lambda_T^*(t)\ r_T^*(b) \right.$$

$$\left. - (-1)^T \frac{\sqrt{2}}{2} \eta^*(t)\ y_c^*(b) - \frac{\sqrt{2}}{2} \mu^*(t)\ z_c^*(b) \right]^2$$

$$+ \sum_{T=3,4,7,8} \frac{1}{2} k_{bTr} \left\{ \lambda_T^*(t)\ r_T^*(b) - \frac{1}{2}\left[(-1)^T + 1\right] \eta^*(t)\ y_c^*(b) \right.$$

$$\left. + \frac{1}{2}\left[(-1)^T - 1\right] \mu^*(t)\ z_c^*(b) \right\}^2 + \sum_{T=1,2,5,6} \frac{1}{2} k_{rTr} \left[\lambda_T^*(t)\ r_T^{*\prime}(b) \right.$$

$$\left. - (-1)^T \frac{\sqrt{2}}{2} \eta^*(t)\ y_c^{*\prime}(b) - \frac{\sqrt{2}}{2} \mu^*(t)\ z_c^{*\prime}(b) \right]^2$$

$$+ \sum_{T=3,4,7,8} \frac{1}{2} k_{rTr} \left\{ \lambda_T^*(t)\ r_T^{*\prime}(b) - \frac{1}{2}\left[(-1)^T + 1\right] \eta^*(t)\ y_c^{*\prime}(b) \right.$$

$$\left. + \frac{1}{2}\left[(-1)^T - 1\right] \mu^*(t)\ z_c^{*\prime}(b) \right\}^2 + \sum_{T=1,2,5,6} \frac{1}{2} k_{aTt} \left[\sigma_T^*(t)\ t_T^*(a) \right.$$

$$+ (-1)^T \frac{\sqrt{2}}{2} \eta^*(t)\ y_c^*(a) - \frac{\sqrt{2}}{2} \mu^*(t)\ z_c^*(a) + \zeta(T)\ R\nu^*(t)\ \psi^*(a)\Bigg]^2$$

$$+ \sum_{T=3,4,7,8} \frac{1}{2} k_{aTt} \Bigg\{\sigma_T^*(t)\ t_T^*(a) + \frac{1}{2}\left[(-1)^T - 1\right] \eta^*(t)\ y_c^*(a)$$

$$- \frac{1}{2}\left[(-1)^T + 1\right] \mu^*(t)\ z_c^*(a) + \zeta(T)\ R\nu^*(t)\ \psi^*(a)\Bigg\}^2$$

$$+ \sum_{T=1,2,5,6} \frac{1}{2} k_{bTt} \Bigg[\sigma_T^*(t)\ t_T^*(b) + (-1)^T \frac{\sqrt{2}}{2} \eta^*(t)\ y_c^*(b)$$

$$- \frac{\sqrt{2}}{2} \mu^*(t)\ z_c^*(b) + \zeta(T)\ R\nu^*(t)\ \psi^*(b)\Bigg]^2$$

$$+ \sum_{T=3,4,7,8} \frac{1}{2} k_{bTt} \Bigg\{\sigma_T^*(t)\ t_T^*(b) + \frac{1}{2}\left[(-1)^T - 1\right] \eta^*(t)\ y_c^*(b)$$

$$- \frac{1}{2}\left[(-1)^T + 1\right] \mu^*(t)\ z_c^*(b) + \zeta(T)\ R\nu^*(t)\ \psi^*(b)\Bigg\}^2$$

$$+ \sum_{T=1,2,5,6} \frac{1}{2} k_{rTt} \left[\sigma_T^*(t)\ t_T^{*\prime}(b) + (-1)^T \frac{\sqrt{2}}{2} \eta^*(t)\ y_c^{*\prime}(b) - \frac{\sqrt{2}}{2} \mu^*(t)\ z_c^{*\prime}(b)\right]^2$$

$$+ \sum_{T=3,4,7,8} \frac{1}{2} k_{rTt} \Bigg\{\sigma_T^*(t)\ t_T^{*\prime}(b) + \frac{1}{2}\left[(-1)^T - 1\right] \eta^*(t)\ y_c^{*\prime}(b) +$$

$$- \frac{1}{2}\left[(-1)^T + 1\right] \mu^*(t)\ z_c^{*'}(b)\Big\}^2 + \frac{1}{2}\ \eta^{*2}(t)\ M_{GY}\ \omega^2_{FFY}$$

$$+ \frac{1}{2}\ \mu^{*2}(t)\ M_{GZ}\ \omega^2_{FFZ} + \frac{1}{2}\ \nu^{*2}(t)\ I_{G\psi}\ \omega^2_{FF\psi}$$

$$+ \sum_{T=1}^{8} \frac{1}{2}\ \lambda_T^{*2}(t)\ M_{GTr}\ \omega^2_{HHTr} + \sum_{T=1}^{8} \frac{1}{2}\ \sigma_T^{*2}(t)\ M_{GTt}\ \omega^2_{HHTt}.$$

The equations of motion were obtained by applying Lagrangé's equations, where

$$L = K.E. - P.E.$$

and

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0.$$

This procedure gives a system of i equations in i generalized coordinates. Thus,

$$\left\{q_i\right\} = \begin{bmatrix} \eta^*(t) \\ \mu^*(t) \\ \nu^*(t) \\ \lambda_1^*(t) \\ \cdot \\ \cdot \\ \cdot \\ \lambda_8^*(t) \\ \sigma_1^*(t) \\ \cdot \\ \cdot \\ \cdot \\ \sigma_8^*(t) \end{bmatrix}, \quad \text{the generalized coordinates.}$$

The following examples will serve to illustrate the procedure for determining the generalized coordinates, q_i.

First example, $q = \lambda_1^*(t)$:

$$\frac{d}{dt}\left[\frac{\partial(K.E.)}{\partial\dot{\lambda}_1^*(t)}\right] = \int_{\ell_1} \bar{m}_1(x)\ \ddot{\lambda}_1^*(t)\ r_1^{*2}(x)\ dx = -\ \omega^2\ \lambda_1^*(t) \int_{\ell_1} \bar{m}_1(x)\ r_1^{*2}(x)\ dx$$

$$= -\ \omega^2 \lambda_1^*(t)\ M_{G1r}$$

$$\frac{\partial(P.E.)}{\partial\lambda_1^*(t)} = k_{a1r}\left[\lambda_1^*(t)\ r_1^*(a) + \frac{\sqrt{2}}{2}\ \eta^*(t)\ y_c^*(a) - \frac{\sqrt{2}}{2}\ \mu^*(t)\ z_c^*(a)\right] r_1^*(a)$$

$$+\ k_{b1r}\left[\lambda_1^*(t)\ r_1^*(b) + \frac{\sqrt{2}}{2}\ \eta^*(t)\ y_c^*(b) - \frac{\sqrt{2}}{2}\ \mu^*(t)\ z_c^*(b)\right] r_1^*(b)$$

$$+\ k_{r1r}\left[\lambda_1^*(t)\ r_1^{*\prime}(b) + \frac{\sqrt{2}}{2}\ \eta^*(t)\ y_c^{*\prime}(b) - \frac{\sqrt{2}}{2}\ \mu^*(t)\ z_c^{*\prime}(b)\right] r_1^{*\prime}(b)$$

$$+\ \lambda_1^*(t)\ M_{G1r}\ \omega_{HH1r}^2.$$

Second example, $q = \nu^*(t)$:

$$\frac{d}{dt}\left[\frac{\partial(K.E.)}{\partial\dot{\nu}^*(t)}\right] = \int_{\ell_c} \bar{I}_{\nu c}(x)\ \ddot{\nu}^*(t)\psi^{*2}(x)dx = -\ \omega^2\nu^*(t) \int_{\ell_c} \bar{I}_{\nu c}(x)\ \psi^{*2}(x)\ dx$$

$$= -\ \omega^2\ \nu^*(t)\ I_{G\psi}$$

$$\frac{\partial(\text{P.E.})}{\partial\nu^*(t)} = k_\psi\ \nu^*(t)\ \psi^{*2}(s) + k_{a1t}\left[\sigma_1^*(t)\ t_1^*(a) - \frac{\sqrt{2}}{2}\ \eta^*(t)\ y_c^*(a)\right.$$

$$\left. - \frac{\sqrt{2}}{2}\ \mu^*(t)\ z_c^*(a) + R\nu^*(t)\ \psi^*(a)\right]\left[R\psi^*(a)\right]$$

$$+ \ldots + k_{a8t}\left[\sigma_8^*(t)\ t_8^*(a) - \mu^*(t)\ z_c^*(a) + R\nu^*(t)\ \psi^*(a)\right]\left[R\psi^*(a)\right]$$

$$+ k_{b1t}\left[\sigma_1^*(t)\ t_1^*(b) - \frac{\sqrt{2}}{2}\ \eta^*(t)\ y_c^*(b) - \frac{\sqrt{2}}{2}\ \mu^*(t)\ z_c^*(b)\right.$$

$$\left. + R\nu^*(t)\ \psi^*(b)\right]\left[R\psi^*(b)\right]$$

$$+ \ldots + k_{b8t}\left[\sigma_8^*(t)\ t_8^*(b) - \mu^*(t)\ z_c^*(b) + R\nu^*(t)\ \psi^*(b)\right]\left[R\psi^*(b)\right]$$

$$+ \nu^*(t)\ I_{G\psi}\ \omega_{FF\psi}^2.$$

For a symmetric system, note that the following equalities apply:

1. $\lambda_1^*(t) = \lambda_5^*(t)$, $\lambda_2^*(t) = \lambda_6^*(t)$, $\lambda_3^*(t) = \lambda_7^*(t)$, $\lambda_4^*(t) = \lambda_8^*(t)$
2. $k_{a1r} = k_{a5r}$, etc.
3. $k_{b1r} = k_{b5r}$, etc.
4. $k_{r1r} = k_{r5r}$, etc.
5. $k_{a1t} = k_{a5t}$, etc.
6. $k_{b1t} = k_{b5t}$, etc.
7. $k_{r1t} = k_{r5t}$, etc.

To combine terms, the following definitions were made:

1. $\frac{1}{2} k_{a1r} + \frac{1}{2} k_{a2r} + k_{a4r} + \frac{1}{2} k_{a1t} + \frac{1}{2} k_{a2t} + k_{a3t} = \frac{1}{2} K_{A\eta}$

2. $\frac{1}{2} k_{b1r} + \frac{1}{2} k_{b2r} + k_{b4r} + \frac{1}{2} k_{b1t} + \frac{1}{2} k_{b2t} + k_{b3t} = \frac{1}{2} K_{B\eta}$

3. $\frac{1}{2} k_{r1r} + \frac{1}{2} k_{r2r} + k_{r4r} + \frac{1}{2} k_{r1t} + \frac{1}{2} k_{r2t} + k_{r3t} = \frac{1}{2} K_{R\eta}$

4. $\frac{1}{2} k_{a1r} + \frac{1}{2} k_{a2r} + k_{a3r} + \frac{1}{2} k_{a1t} + \frac{1}{2} k_{a2t} + k_{a4t} = \frac{1}{2} K_{A\mu}$

5. $\frac{1}{2} k_{b1r} + \frac{1}{2} k_{b2r} + k_{b3r} + \frac{1}{2} k_{b1t} + \frac{1}{2} k_{b2t} + k_{b4t} = \frac{1}{2} K_{B\mu}$

6. $\frac{1}{2} k_{r1r} + \frac{1}{2} k_{r2r} + k_{r3r} + \frac{1}{2} k_{r1t} + \frac{1}{2} k_{r2t} + k_{r4t} = \frac{1}{2} K_{R\mu}$

7. $k_{a1t} + k_{a2t} + k_{a3t} + k_{a4t} = K_{TA\psi}$

8. $k_{b1t} + k_{b2t} + k_{b3t} + k_{b4t} = K_{TB\psi}$

Because of symmetry, there are alternate expressions for the above K terms. For example, the first expression can be written as follows:

1a. $\frac{1}{2} k_{a5r} + \frac{1}{2} k_{a6r} + k_{a8r} + \frac{1}{2} k_{a5t} + \frac{1}{2} k_{a6t} + k_{a7t} = \frac{1}{2} K_{A\eta}$.

Similar expressions can be written for 1/2 $(K_{B\eta})$, etc. By means of the above expressions, the system of simultaneous equations was reduced to a system with the following generalized coordinates:

$$\left\{\bar{q}_i\right\} = \begin{bmatrix} \eta^*(t) \\ \mu^*(t) \\ \lambda_1^*(t) \\ \cdot \\ \cdot \\ \cdot \\ \lambda_4^*(t) \\ \sigma_1^*(t) \\ \cdot \\ \cdot \\ \cdot \\ \sigma_8^*(t) \\ \nu^*(t) \end{bmatrix} .$$

The following examples show typical determinations of the generalized coordinates, $\bar{q}_i$.

First example, $q = \lambda_1^*(t)$:

$$\eta^*(t) \left[\frac{\sqrt{2}}{2} k_{r1r}\, y_c^{*'}(b)\, r_1^{*'}(b) + \frac{\sqrt{2}}{2} k_{a1r}\, y_c^*(a)\, r_1^*(a)\right.$$

$$\left. + \frac{\sqrt{2}}{2} k_{b1r}\, y_c^*(b)\, r_1^*(b)\right]$$

$$+ \mu^*(t) \left[- \frac{\sqrt{2}}{2} k_{r1r}\, z_c^{*'}(b)\, r_1^{*'}(b) - \frac{\sqrt{2}}{2} k_{a1r}\, z_c^*(a)\, r_1^*(a)\right.$$

$$\left. - \frac{\sqrt{2}}{2} k_{b1r}\, z_c^*(b)\, r_1^*(b)\right]$$

$$+ \lambda_1^*(t) \left[M_{G1r} (\omega^2_{HH1r} - \omega^2) + k_{r1r} r_1^{*'^2}(b) + k_{a1r} r_1^{*^2}(a) + k_{b1r} r_1^{*^2}(b) \right]$$

$$+ \lambda_2^*(t) [0] + \ldots + \lambda_4^*(t) [0] + \sigma_1^*(t) [0] + \ldots + \sigma_8^*(t) [0]$$

$$+ \nu^*(t) [0] = 0.$$

Second example, $q = \nu^*(t)$:

$$\eta^*(t) [0] + \mu^*(t) [0] + \lambda_1^*(t) [0] + \ldots + \lambda_4^*(t) [0]$$

$$+ \sigma_1^*(t) \left[k_{a1t} t_1^*(a) R\psi^*(a) + k_{b1t} t_1^*(b) R\psi^*(b) \right] + \ldots$$

$$+ \sigma_8^*(t) \left[k_{a8t} t_8^*(a) R\psi^*(a) + k_{b8t} t_8^*(b) R\psi^*(b) \right]$$

$$+ \nu^*(t) \left[I_{G\psi} (\omega^2_{FF\psi} - \omega^2) + 2k_{TA\psi} R^2\psi^{*^2}(a) + 2k_{TB\psi} R^2\psi^{*^2}(b) \right.$$

$$\left. + k_\psi \psi^{*^2}(s) \right] = 0.$$

The generalized coordinates $\{\bar{q}_i\}$ were then transformed by algebraic operations into new generalized coordinates $\{\bar{\bar{q}}_i\}$. In these new coordinates, the equations of motion possess no coupling between center tank pitch, yaw, and torsion. These new generalized coordinates are as follows:

$$
\left\{\bar{\bar{q}}_i\right\} = \begin{bmatrix}
\eta^*(t) \\
\lambda_1^*(t) - \lambda_2^*(t) \\
\lambda_4^*(t) \\
\frac{1}{2}\,[\sigma_1^*(t) - \sigma_2^*(t) + \sigma_5^*(t) + \sigma_6^*(t)] \\
\frac{1}{2}\,[\sigma_3^*(t) + \sigma_7^*(t)] \\
\hdashline
\mu^*(t) \\
-\,[\lambda_1^*(t) + \lambda_2^*(t)] \\
\lambda_3^*(t) \\
\frac{1}{2}\,[\sigma_1^*(t) + \sigma_2^*(t) + \sigma_5^*(t) + \sigma_6^*(t)] \\
\frac{1}{2}\,[\sigma_4^*(t) + \sigma_8^*(t)] \\
\hdashline
\nu^*(t) \\
\frac{1}{2}\,[\sigma_1^*(t) - \sigma_2^*(t) - \sigma_5^*(t) + \sigma_6^*(t)] \\
\frac{1}{2}\,[\sigma_3^*(t) - \sigma_4^*(t) - \sigma_7^*(t) + \sigma_8^*(t)] \\
\hdashline
[\sigma_1^*(t) + \sigma_2^*(t) - \sigma_5^*(t) - \sigma_6^*(t)] \\
[\sigma_3^*(t) + \sigma_4^*(t) - \sigma_7^*(t) - \sigma_8^*(t)]
\end{bmatrix}
$$

In terms of the coordinates $\{\bar{\bar{q}}_i\}$, the system was reduced to five independent sets of simultaneous equations which were solved by existing computer eigenvector programs. Expressed in eigenvector form, the equations of motion are:

$$\begin{bmatrix} [A] & [0] & [0] & [0] & [0] \\ [0] & [B] & [0] & [0] & [0] \\ [0] & [0] & [C] & [0] & [0] \\ [0] & [0] & [0] & [D] & [0] \\ [0] & [0] & [0] & [0] & [E] \end{bmatrix} \left[\left\{ \bar{\bar{q}}_i \right\} \right] = \left[0 \right].$$

Some additional identities for the symmetric vehicle were defined where the subscripts 0 and F refer to LOX and fuel, respectively.

1. $k_{r1r} = k_{r2r} = k_{ror}$

2. $k_{a1r} = k_{a2r} = k_{aor}$

3. $k_{b1r} = k_{b2r} = k_{bor}$

4. $k_{r3r} = k_{r4r} = k_{rFr}$

5. $k_{a3r} = k_{a4r} = k_{aFr}$.

6. $k_{b3r} = k_{b4r} = k_{bFr}$

7. $k_{r1t} = k_{r2t} = k_{r5t} = k_{r6t} = k_{rot}$

8. $k_{a1t} = k_{b1t} = k_{a5t} = k_{a6t} = k_{aot}$

9. $k_{b1t} = k_{b2t} = k_{b5t} = k_{b6t} = k_{bot}$

10. $k_{r3t} = k_{r4t} = k_{r7t} = k_{r8t} = k_{rFt}$

11. $k_{a3t} = k_{a4t} = k_{a7t} = k_{a8t} = k_{aFt}$

12. $k_{b3t} = k_{b4t} = k_{b7t} = k_{b8t} = k_{bFt}$

The same subscript identification was used for the quantities $r_o^*(x)$, $r_F^*(x)$, $t_o^*(x)$, $t_F^*(x)$, M_{Gor}, M_{GFr}, M_{Got}, M_{GFt}, ω_{HHor}, ω_{HHFr}, ω_{HHot}, ω_{HHFt}.

The uncoupled equations for the submatrices are as follows.

1. A submatrix, $\eta^*(t)$ uncoupled equation:

$$\eta^*(t) \Big\{ M_{GY} \left(\omega_{FFY}^2 - \omega^2\right) + K_{R\eta}\, y_c^{*'2}(a) + K_{A\eta}\, y_c^{*2}(a) + K_{B\eta}\, y_c^{*2}(b) + K_{SJ}\, y_c^{*2}(s)$$

$$+ K_{SRJ}\, y_c^{*'2}(s) + K_{LJ} \left[y_c^{*'}(a) - y_c^{*'}(b) \right]^2 \Big\} +$$

$$+\left(\lambda_1^*(t) - \lambda_2^*(t)\right)\left[\sqrt{2}\ k_{ror}\ y_c^{*\prime}(b)\ r_o^{*\prime}(b) + \sqrt{2}\ k_{aor}\ y_c^*(a)\ r_o^*(a)\right.$$

$$\left. + \sqrt{2}\ k_{bor}\ y_c^*(b)\ r_o^*(b)\right] + \lambda_4^*(t)\left[- 2k_{rFr}\ y_c^{*\prime}(b)\ r_F^{*\prime}(b)\right.$$

$$\left. - 2k_{aFr}\ y_c^*(a)\ r_F^*(a) - 2k_{bFr}\ y_c^*(b)\ r_F^*(b)\right]$$

$$+ \frac{1}{2}\left[\sigma_1^*(t) - \sigma_2^*(t) + \sigma_5^*(t) - \sigma_6^*(t)\right]\left[- \sqrt{2}\ k_{rot}\ y_c^{*\prime}(b)\ t_o^{*\prime}(b)\right.$$

$$\left. - \sqrt{2}\ k_{aot}\ y_c^*(a)\ t_o^*(a) - \sqrt{2}\ k_{bot}\ y_c^*(b)\ t_o^*(b)\right]$$

$$+ \frac{1}{2}\left[\sigma_3^*(t) + \sigma_7^*(t)\right]\left[- 2k_{rFt}\ y_c^{*\prime}(b)\ t_F^{*\prime}(b) - 2k_{aFt}\ y_c^*(a)\ t_F^*(a)\right.$$

$$\left. - 2k_{bFt}\ y_c^*(b)\ t_F^*(b)\right] = 0.$$

2. A submatrix, $[\lambda_1^*(t) - \lambda_2^*(t)]$ uncoupled equation:

$$\eta^*(t)\left[\sqrt{2}\ k_{ror}\ y_c^{*\prime}(b)\ r_o^{*\prime}(b) + \sqrt{2}\ k_{aor}\ y_c^*(a)\ r_o^*(a)\right.$$

$$\left. + \sqrt{2}\ k_{bor}\ y_c^*(b)\ r_o^*(b)\right] + \left[\lambda_1^*(t) - \lambda_2^*(t)\right]\left[M_{Gor}\ (\omega_{HHor} - \omega^2)\right.$$

$$\left. + k_{ror}\ r_o^{*\prime 2}(b) + k_{aor}\ r_o^{*2}(a) + k_{bor}\ r_o^{*2}(b)\right] = 0.$$

3. A submatrix, $\lambda_4^*(t)$ uncoupled equation:

$$\eta^*(t)\left[-\,2k_{rFr}\,y_c^{*\prime}(b)\,r_F^{*\prime}(b)\,-\,2k_{aFr}\,y_c^*(a)\,r_F^*(a)\,-\,2k_{bFr}\,y_c^*(b)\,r_F^*(b)\right]$$

$$+\,\lambda_4^*(t)\left[2M_{GFr}\,(\omega_{HHFr}^2-\omega^2)+2k_{rFr}\,r_F^{*\prime 2}(b)+2k_{aFr}\,r_F^{*2}(a)\right.$$

$$\left.+\,2k_{bFr}\,r_F^{*2}(b)\right]=0.$$

4. A submatrix, $\frac{1}{2}\,[\sigma_1^*(t)-\sigma_2^*(t)+\sigma_5^*(t)-\sigma_6^*(t)]$ uncoupled equation:

$$\eta^*(t)\left[-\sqrt{2}\;k_{rot}\,y_c^{*\prime}(b)\,t_o^{*\prime}(b)-\sqrt{2}\;k_{aot}\,y_c^*(a)\,t_o^*(a)\right.$$

$$\left.-\sqrt{2}\;k_{bot}\,y_c^*(b)\,t_o^*(b)\right]+\frac{1}{2}\left[\sigma_1^*(t)-\sigma_2^*(t)+\sigma_5^*(t)-\sigma_6^*(t)\right]$$

$$\cdot\left[M_{Got}\,(\omega_{HHot}^2-\omega^2)+k_{rot}\,t_o^{*\prime 2}(b)+k_{aot}\,t_o^{*2}(a)+k_{bot}\,t_o^{*2}(b)\right]=0.$$

5. A submatrix, $\frac{1}{2}\,[\sigma_3^*(t)+\sigma_7^*(t)]$ uncoupled equation:

$$\eta^*(t)\left[-\,2k_{rFt}\,y_c^{*\prime}(b)\,t_F^{*\prime}(b)-2k_{aFt}\,y_c^*(a)\,t_F^*(a)-2k_{bFt}\,y_c^*(b)\,t_F^*(b)\right]$$

$$+\,\frac{1}{2}\left[\sigma_3^*(t)+\sigma_7^*(t)\right]\left[2M_{GFt}\,(\omega_{HHFt}^2-\omega^2)+2k_{rFt}\,t_F^{*\prime 2}(b)\right.$$

$$\left.+\,2k_{aFt}\,t_F^{*2}(a)+2k_{bFt}\,t_F^{*2}(b)\right]=0.$$

6. B submatrix, $\mu^*(t)$ uncoupled equation:

$$\mu^*(t)\left\{M_{GZ}\,(\omega^2_{FFZ} - \omega^2) + K_{R\mu}\, z^{*'^2}_c(b) + K_{A\mu}\, z^{*^2}_c(a) + K_{B\mu}\, z^{*^2}_c(b) + K_{SK}\, z^{*^2}_c(s)\right.$$

$$\left. + K_{SRK}\, z^{*'^2}_c(s) + K_{LK}\left[z^{*'}_c(a) - z^{*'}_c(b)\right]^2\right\}$$

$$- \left[\lambda^*_1(t) + \lambda^*_2(t)\right]\left[\sqrt{2}\; k_{ror}\; z^{*'}_c(b)\; r^{*'}_o(b) + \sqrt{2}\; k_{aor}\; z^*_c(a)\; r^*_o(a)\right.$$

$$\left. + \sqrt{2}\; k_{bor}\; z^*_c(b)\; r^*_o(b)\right] + \lambda^*_3(t)\left[- 2k_{rFr}\; z^{*'}_c(b)\; r^{*'}_F(b)\right.$$

$$\left. - 2k_{aFr}\; z^*_c(a)\; r^*_F(a) - 2k_{bFr}\; z^*_c(b)\; r^*_F(b)\right]$$

$$+ \frac{1}{2}\left[\sigma^*_1(t) + \sigma^*_2(t) + \sigma^*_5(t) + \sigma^*_6(t)\right]\left[- \sqrt{2}\; k_{rot}\; z^{*'}_c(b)\; t^{*'}_o(b)\right.$$

$$\left. - \sqrt{2}\; k_{aot}\; z^*_c(a)\; t^*_o(a) - \sqrt{2}\; k_{bot}\; z^*_c(b)\; t^*_o(b)\right]$$

$$+ \frac{1}{2}\left[\sigma^*_4(t) + \sigma^*_8(t)\right]\left[- 2k_{rFt}\; z^{*'}_c(b)\; t^{*'}_F(b) - 2k_{aFt}\; z^*_c(a)\; t^*_F(a)\right.$$

$$\left. - 2k_{bFt}\; z^*_c(b)\; t^*_F(b)\right] = 0.$$

7. B submatrix, $-[\lambda_1^*(t) + \lambda_2^*(t)]$ uncoupled equation:

$$\mu^*(t)\Big[\sqrt{2}\ k_{ror}\ z_c^{*\prime}(b)\ r_o^{*\prime}(b) + \sqrt{2}\ k_{aor}\ z_c^*(t)\ r_o^*(a)$$

$$+ \sqrt{2}\ k_{bor}\ z_c^*(b)\ r_o^*(b)\Big] - \Big[\lambda_1^*(t) + \lambda_2^*(t)\Big]\Big[M_{Gor}\ (\omega^2_{HHor} - \omega^2)$$

$$+ k_{ror}\ r_o^{*\prime 2}(b) + k_{aor}\ r_o^{*2}(a) + k_{bor}\ r_o^{*2}(b)\Big] = 0.$$

8. B submatrix, $\lambda_3^*(t)$ uncoupled equation:

$$\mu^*(t)\ \Big[-2k_{rFr}\ z_c^{*\prime}(b)\ r_F^{*\prime}(b) - 2k_{aFr}\ z_c^*(a)\ r_F^*(a) - 2k_{bFr}\ z_c^*(b)\ r_F^*(b)\Big]$$

$$+ \lambda_3^*(t)\ \Big[2M_{GFr}\ (\omega^2_{HHFr} - \omega^2) + 2k_{rFr} r_F^{*\prime 2}(b) + 2k_{aFr}\ r_F^{*2}(a)$$

$$+ 2k_{bFr}\ r_F^{*2}(b)\Big] = 0.$$

9. B submatrix, $\frac{1}{2}\ [\sigma_1^*(t) + \sigma_2^*(t) + \sigma_5^*(t) + \sigma_6^*(t)]$ uncoupled equation:

$$\mu^*(t)\ \Big[-\sqrt{2}\ k_{rot}\ z_c^{*\prime}(b)\ t_o^{*\prime}(b) - \sqrt{2}\ k_{aot}\ z_c^*(a)\ t_o^*(a)$$

$$- \sqrt{2}\ k_{bot}\ z_c^*(b)\ t_o^*(b)\Big] + \frac{1}{2}\Big[\sigma_1^*(t) + \sigma_2^*(t) + \sigma_5^*(t) + \sigma_6^*(t)\Big]$$

$$\cdot \Big[M_{Got}\ (\omega^2_{HHot} - \omega^2) + k_{rot}\ t_o^{*\prime 2}(b) + k_{aot}\ t_o^{*2}(a) + k_{bot}\ t_o^{*2}(b)\Big] = 0.$$

10. B submatrix, $\frac{1}{2}\,[\sigma_4^*(t) + \sigma_8^*(t)]$ uncoupled equation:

$$\mu^*(t)\left[- 2k_{rFt}\; z_c^{*\prime}(b)\; t_F^{*\prime}(b) - 2k_{aFt}\; z_c^*(a)\; t_F^*(a) - 2k_{bFt}\; z_c^*(b)\; t_F^*(b)\right]$$

$$+ \frac{1}{2}\left[\sigma_4^*(t) + \sigma_8^*(t)\right]\left[2M_{GFt}\;(\omega_{HHFt}^2 - \omega^2) + 2k_{rFt}\; t_F^{*\prime 2}(b)\right.$$

$$\left.+ 2k_{aFt}\; t_F^{*2}(a) + 2k_{bFt}\; t_F^{*2}(b)\right] = 0.$$

11. C submatrix, $\nu^*(t)$ uncoupled equation:

$$\nu^*(t)\left[I_{G\psi}\;(\omega_{FF\psi}^2 - \omega^2) + 2K_{TA\psi}\; R^2\psi^{*2}(a) + 2K_{TB\psi}\; R^2\psi^{*2}(b) + k_\psi\; \psi^{*2}(s)\right]$$

$$+ \frac{1}{2}\left[\sigma_1^*(t) - \sigma_2^*(t) - \sigma_5^*(t) + \sigma_6^*(t)\right]\left[2k_{aot}\; t_o^*(a)\; R\psi^*(a)\right.$$

$$\left.+ 2k_{bot}\; t_o^*(b)\; R\psi^*(b)\right] + \frac{1}{2}\left[\sigma_3^*(t) - \sigma_4^*(t) - \sigma_7^*(t) + \sigma_8^*(t)\right]$$

$$\cdot\left[2k_{aFt}\; t_F^*(a)\; R\psi^*(b) + 2k_{bFt}\; t_F^*(b)\; R\psi^*(b)\right] = 0.$$

12. C submatrix, $\frac{1}{2}\,[\sigma_1^*(t) - \sigma_2^*(t) - \sigma_5^*(t) + \sigma_6^*(t)]$ uncoupled equation:

$$\nu^*(t)\left[2k_{aot}\; t_o^*(a)\; R\psi^*(a) + 2k_{bot}\; t_o^*(b)\; R\psi^*(b)\right],$$

$$+ \frac{1}{2}\left[\sigma_1^*(t) - \sigma_2^*(t) - \sigma_5^*(t) + \sigma_6^*(t)\right]\left[M_{Got}\ (\omega^2_{HHot} - \omega^2)\right.$$

$$\left. + k_{rot}\ t_o^{*'^2}(b) + k_{aot}\ t_o^{*^2}(a) + k_{bot}\ t_o^{*^2}(b)\right] = 0.$$

13. C submatrix, $\frac{1}{2}\,[\sigma_3^*(t) - \sigma_4^*(t) - \sigma_7^*(t) + \sigma_8^*(t)]$ uncoupled equation:

$$\nu^*(t)\left[2k_{aFt}\ t_F^*(a)\ R\psi^*(a) + 2k_{bFt}\ t_F^*(b)\ R\psi^*(b)\right]$$

$$+ \frac{1}{2}\left[\sigma_3^*(t) - \sigma_4^*(t) - \sigma_7^*(t) + \sigma_8^*(t)\right]\left[M_{GFt}\ (\omega^2_{HHFt} - \omega^2)\right.$$

$$\left. + k_{rFt}\ t_F^{*'^2}(b) + k_{aFt}\ t_F^{*^2}(a) + k_{bFt}\ t_F^{*^2}(b)\right] = 0.$$

14. D submatrix, $[\sigma_1^*(t) + \sigma_2^*(t) - \sigma_5^*(t) - \sigma_6^*(t)]$ uncoupled equation:

$$\left[\sigma_1^*(t) + \sigma_2^*(t) - \sigma_5^*(t) - \sigma_6^*(t)\right]\left[M_{Got}\ (\omega^2_{HHot} - \omega^2)\right.$$

$$\left. + k_{rot}\ t_o^{*'^2}(b) + k_{aot}\ t_o^{*^2}(a) + k_{bot}\ t_o^{*^2}(b)\right] = 0.$$

15. E submatrix, $[\sigma_3^*(t) + \sigma_4^*(t) - \sigma_7^*(t) - \sigma_8^*(t)]$ uncoupled equations:

$$\left[\sigma_3^*(t) + \sigma_4^*(t) - \sigma_7^*(t) - \sigma_8^*(t)\right]\left[M_{GFt}\,(\omega^2_{HHFt} - \omega^2)\right.$$

$$\left. + k_{rFt}\, t_F^{*'}{}^2(a) + k_{aFt}\, t_F^{*2}(a) + k_{bFt}\, t_F^{*2}(b)\right] = 0.$$

IV. RESULTS

The most interesting result of this analysis is shown in the existence of a set of generalized coordinates in which there is no coupling in the equations of motion for center tank pitch, yaw, and torsion. The multiple beam analysis (see reference) which has been the basis for the Aero-Astrodynamics Laboratory computations of Saturn I and IB frequencies and mode shapes is based on the implicit assumption of such uncoupling. The analysis presented here establishes the validity of this assumption.

The A submatrix, a seventeen-degrees-of-freedom system, was solved using an eigenvalue iteration procedure. Input data were provided for the Saturn SA-5 vehicle for 35 seconds flight time. A comparable analysis was made using the basic multiple beam program described in the reference. A comparison of the results of the basic program with those obtained using the A submatrix of the analysis presented in this report is shown in Table 1. Good correlation was obtained for this system.

Two special types of nonsymmetry were analyzed in a limited parameter study. By consideration of particular nonsymmetry conditions along the principal axes of the coupled system, the set of simultaneous equations was preserved in the uncoupled form. The effects of nonsymmetry were reflected thereby only as changes in the constant coefficients supplied as input data to the computations. The types of nonsymmetry considered were changes in the elastic characteristics of the outer tanks and in the elastic support of the outer tanks. Results of comparisons of the symmetrical vehicle response with that of the unsymmetrical vehicle are given in Table 1.

As can be seen from this comparison, the effects of these types of nonsymmetry are relatively small. For example, the maximum change in frequency caused by a 21 percent change in fuel tank stiffness is about 1.9 percent. This occurs in the fourth mode, which is characterized as a fuel tank tangential mode. Differences of the same order exist between the results of the basic multiple beam analysis and the present analysis. These differences are insignificant. Despite the small magnitude of frequency changes, the analysis shows the effects of such nonsymmetry qualitatively. For example, it can be seen that increasing the fuel tank stiffness increases all seven natural frequencies tabulated.

The results presented here do not treat all of the cases of interest; however, computer program limitations restricted this study to these simple cases. When this deficiency is corrected, more complex cases of nonsymmetry will be treated and the results reported.

TABLE 1

Mode	Mode Description	Frequency (radians/sec) Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
1	FTR	13.29	13.29	13.30	13.28	13.29	13.29
2	LTR	15.17	15.21	15.35	15.02	15.21	15.21
3	CT1	15.77	16.18	16.20	16.15	16.18	16.18
4	FTT	17.92	17.96	18.30	17.67	17.96	17.96
5	LTT	26.26	26.57	26.85	26.33	26.57	26.57
6	CT2	35.00	35.09	35.11	35.07	35.09	35.09
7	CT3	64.61	64.66	64.66	64.66	64.66	64.66

Case	Description
1	basic multiple beam theory
2	subject analysis for symmetrical vehicle
3	subject analysis, fuel tank stiffness up 21 percent
4	subject analysis, fuel tank stiffness down 21 percent
5	subject analysis, fuel tank support stiffness up 10 percent
6	subject analysis, fuel tank support stiffness down 10 percent

Abbreviation	Mode Description
FTR	fuel tank radial
LTR	LOX tank radial
CT1 (2,3)	center tank first (second, third) mode
FTT	fuel tank tangential
LTT	LOX tank tangential

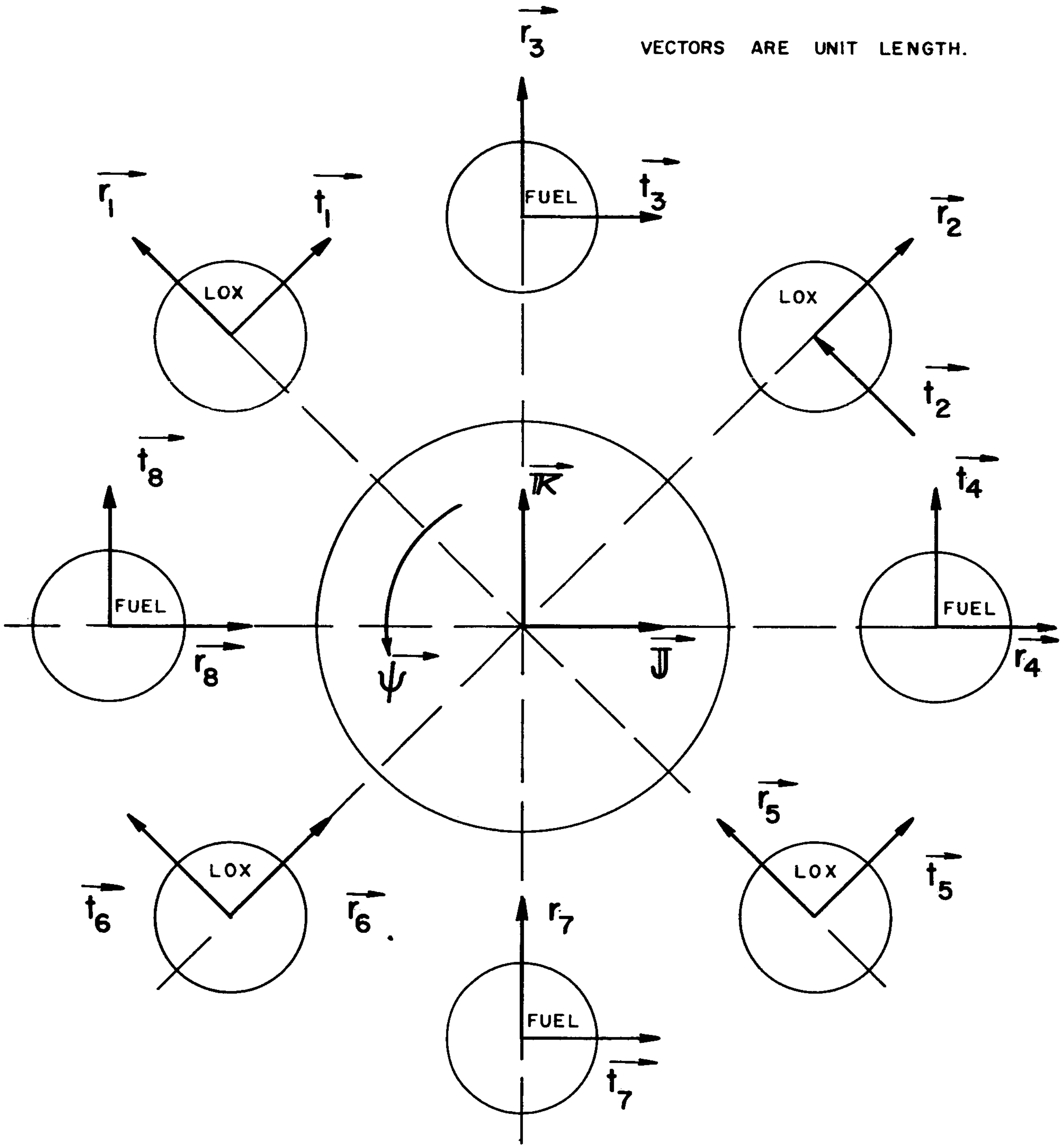

FIGURE 1. COUPLED SYSTEM

REFERENCE

Kiefling, L., "Revised Multiple Beam Vibration Analysis of the Saturn SA-1 Vehicle," MTP-AERO-62-42, May 16, 1962.

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July 16, 1964 APPROVAL NASA TM X-[redacted]

THREE-DIMENSIONAL MULTIPLE BEAM ANALYSIS OF A SATURN I VEHICLE

James L. Milner

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